2018 Annual American Control Conference (ACC)
June 27–29, 2018. Wisconsin Center, Milwaukee, USA

Rendezvous of Wheeled Mobile Robots with Sensing and Power Constraints

Wei Yu[1] and Zhen Kan[2]

Abstract— **This paper considers a rendezvous problem, where a group of wheeled mobile robots with sensing and power constraints is tasked to meet at a common destination. Each robot is modeled in the wheel kinematic level, which is governed by the wheel actuation power constraints. The group of robots is of a leader-follower network structure, where only the leader robot is aware of the desired destination and the follower robots are driven by the leader to the common destination via local sensing of neighboring robots. To ensure the availability of inter-robot communication, the motion of robots is constrained to preserve network connectivity, enabling leader-based guidance via connected paths. In addition, the motion of robots is further constrained to avoid collisions with other robots. To achieve these objectives, a guidance function based decentralized controller is developed to control the wheel velocities of each robot, which ensures global convergence to the destination while guaranteeing the network connectivity, collision avoidance, and not violating its sensing and actuator power capabilities. The simulation results demonstrate the effectiveness of the developed controller.**

I. INTRODUCTION

Networked mobile robots can be deployed to accomplish a wide range of missions, such as foraging, search, rescue and mobile target jamming, etc. To successfully accomplish these tasks, mobile robots are often required to share or exchange collected information by meeting at a desired destination. Therefore, rendezvous problems are of great importance for networked robots in various applications. A successful rendezvous of a networked robot system should satisfy multiple constraints. First, the connectivity of the underlying communication network within the robots should be preserved. Second, no collision within the robots should be allowed. In addition, each robot has its limitations on sensing and maximum wheel velocities due to actuation system power constraints.

The research in [1], [2] has explored the rendezvous problem for a group of autonomous mobile agents to converge to a common point. The synchronized and unsynchronized strategies are developed to drive mobile agents to a single unspecified location by using only position feedback from its sensing regions. A dipolar navigation function was proposed and a discontinuous time-invariant controller was developed for a multi-robot system in [3] to perform nonholonomic navigation for networked robots. The dipolar navigation function is a particular class of potential functions, which was developed from [4] such that the negative gradient field does not have local minima, and the closed-loop navigation function guarantees convergence to the global minimum. The result in [3] was then extended to navigate a nonholonomic system in three dimensions in [5]. A common assumption of aforementioned results is that the network remains connected during the motion evolution, allowing constant interaction between agents. However, the assumption of network connectivity preservation is not always practical. Typically, each agent can only make decisions based on the local information from immediate neighbors within a certain region due to sensing and communication constraints. Since communication/sensing links generally depend on the distance between agents, agent motion may cause the underlying network to disconnect. If the network disconnects, certain agents may no longer be able to communicate and coordinate their motion, leading to the failure of rendezvous.

In order to preserve network connectivity when performing rendezvous task, a circumcenter algorithm is proposed in [6] to avoid the loss of existing links between agents. A potential field-based centralized approach [7] has been developed to ensure the connectivity of a group of agents which requires global knowledge of the complete network structure to determine the control for each robot. A potential field-based distributed approach has been reported in [8] to prevent partitioning the underlying graph by using local information from each agent's immediate neighbors. The advantages of distributed approaches in [8] over centralized approaches in [7] include reduced communication traffic and less demand of advanced hardwares (i.e., Global Positioning System). The result in [9] provides a connectivity-preserving protocol for rendezvous of a discrete-time multi-agent system, and a hybrid dynamic rendezvous protocol in [10] is designed to address finite-time rendezvous problems while preserving network connectivity. One limitation of these research on preserving the network connectivity is that only the point mass model of the robot has been adopted in the control design for the networked robot system without considering the robot nonholonimic characteristics.

A nonholonimic kinematic robot model has been used in [11] to develop a distributed controller for a networked robot system to converge to a destination which are determined from the initial deployment of the robots. In [12], a time-varying centralized controller has been designed based on the nonholonomic kinematic robot model to maintain network connectivity given the condition that the common global destination is known to all the robot. Each robot is treated as an equal role in the group and the undirected interaction

[1]Wei Yu is with Department of Engineering, Massachusetts Maritime Academy, Buzzards Bay, MA, 02532, USA wyu@maritime.edu
[2]Zhen Kan is with Department of Mechanical and Industrial Engineering, University of Iowa, Iowa City, IA, 52242, USA zhen-kan@uiowa.edu

between robots are considered. Different from the controller in [11], the controller in [12] can ensure the networked robot system to rendezvous at any specified destination. Furthermore, a decentralized controller based on the non-holonimic robot model has been designed in [13] using the robot local sensing information of distance and orientation of the neighbors. While the nonholonomic kinematic constraint is considered in [11]–[13], few existing results take into account the actuation constraint, that is, the commanded input may require more actuation than is physically possible by the system.

Based on our previous research in [12]–[14], a real-time decentralized controller is developed in this paper for networked mobile robots to perform rendezvous while considering actuation and network connectivity constraints. Different from [12], [13] in which the robot power constrains are ignored, the robot kinematic model in the wheels level has been adopted to design control laws which can guarantee the control effort does not violate the robot power constraints. While [14] only considers the control of a single robot, this paper considers the control of a networked robot system. The networked robot system is treated as a leader-follower hierarchy. While the leader robots converge to the common destination, the follower robots follow the leader robots through the connectivity tree to perform rendezvous.

II. Problem Formulation

A networked robot system is composed of N mobile robots operating in a workspace $\mathcal{S}$, where $\mathcal{S}$ is a bounded disk area with radius R_w. The workspace $\mathcal{S}$ is divided into a collision-free region S_c and a rendezvous region S_r. All robots have equal actuation capabilities and each robot has sensing and communication limitations encoded by a disk area with radius R, which indicates that two mobile robots can sense and communicate within a distance of R. The networked robot system is modeled as a directed spanning tree $\mathcal{G}(t) = (\mathcal{L}, \mathcal{E}(t))$, where the node set $\mathcal{L} = \{1, \cdots, N\}$ represents the group of robots, and the edge set $\mathcal{E}(t)$ denotes time-varying edges. Every node of the directed spanning tree has one parent except for one node, called the root, and the root node has directed paths to every other node in the tree.

In the networked robot system, a subset of the robots are treated as leader robots which are provided with knowledge of the destination. The other robots are follower robots which can only use local state feedback (i.e., position feedback from immediate neighbors and absolute orientation measurement). The set of leader robots and follower robots are denoted as $\mathcal{L}_L$ and $\mathcal{L}_F$. Since the follower robots are not aware of the destination, they have to stay connected with the leader robots either directly or indirectly through concatenated paths, such that the knowledge of the destination can be delivered to all the nodes through the connected network. Hence, to complete the rendezvous task, maintaining connectivity of the underlying tree is necessary. While the number of leader robots may be chosen arbitrarily for rendezvous task, a single leader robot is focused in this work. The techniques proposed in this work could be extended to the case of multiple leader robots by using containment control.

Let $\mathcal{L}_L = \{1\}$ and $\mathcal{L}_F = \{2, \cdots, N\}$. A directed edge $(j, i) \in \mathcal{E}$ in $\mathcal{G}(t)$ exists between node i and j if their relative distance $d_{ij} \triangleq \|p_i - p_j\| \in \mathbb{R}^+$ is less than R. The directed edge (j, i) indicates that node i is able to access the states (i.e., position and orientation) of node j through local sensing, but not vice versa. Accordingly, node j is a neighbor of node i (also called the parent of node i), and the neighbor set of node i is denoted as $\mathcal{N}_i = \{j \mid (j, i) \in \mathcal{E}\}$, which includes the nodes that can be sensed. The rendezvous region S_r is a bounded disk area with radius R_r centered at the common destination p^*, while the remaining area in $\mathcal{S}$ is the collision-free region S_c. Assume that the workspace $\mathcal{S}$ and the rendezvous region S_r satisfy that $R_w \gg R_r$.

It is assumed that the rendezvous destination p^* and desired orientation θ^* are achievable, which implies that p^* and θ^* do not coincide with some unstable equilibria (i.e., saddle points). The classical rendezvous problem enables the robots to rendezvous at p^* with a desired orientation θ^* in S_r. We address the rendezvous problem by considering additional constraints, including collision avoidance in S_c, network connectivity preservation, and actuation constraints that ensures the commanded input always within the actuator capabilities when performing rendezvous.

III. Robot Wheel Kinematic Model

In this section, the robot wheel kinematic model is discussed, which is constrained by the power of the wheel actuation system.

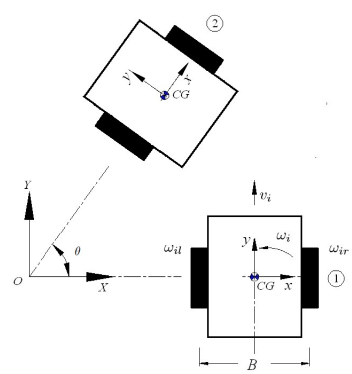

Fig. 1. Robot i moves counterclockwise.

Fig. 1 depicts a robot i moving counterclockwise (CCW) at linear velocity v_i and angular velocity S_i from position 1 to position 2. X–Y denotes the global frame and the body-fixed frame for the robot is given by the x–y. It is assumed that the robot is symmetric and the center of gravity (CG) is at the geometric center. The robot i moves in workspace $\mathcal{S}$ according to the following nonholonomic kinematics:

$$\dot{q}_i = \begin{bmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i(t) \\ \omega_i(t) \end{bmatrix}, \; i = 1, \cdots, N \qquad (1)$$

where $q_i(t) \triangleq \begin{bmatrix} x_i(t) & y_i(t) & \theta_i(t) \end{bmatrix}^T \in \mathbb{R}^3$ denotes the states. $x_i(t)$, $y_i(t)$ and $\theta_i(t)$ are the x, y coordinates and the robot orientation in the X–Y. $v_i(t)$, $\omega_i(t)$ are the linear and angular velocities. The position of robot i is $p_i(t) \triangleq \begin{bmatrix} x_i(t) & y_i(t) \end{bmatrix}^T \in \mathbb{R}^2$.

The robot linear velocity and angular velocity can be expressed in terms of the the angular velocities of the left and right wheels [14], ω_{il} and ω_{ir},

$$\begin{bmatrix} v_i(t) \\ \omega_i(t) \end{bmatrix} = r \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{B} & \frac{1}{B} \end{bmatrix} \begin{bmatrix} \omega_{il}(t) \\ \omega_{ir}(t) \end{bmatrix}, \tag{2}$$

where B is the robot width and r is the wheel radius.

Combining (1) and (2), the robot wheel kinematic model from robot wheels to the robot states is obtained.

$$\dot{q}_i = r \begin{bmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{B} & \frac{1}{B} \end{bmatrix} \begin{bmatrix} \omega_{il}(t) \\ \omega_{ir}(t) \end{bmatrix} \tag{3}$$

The maximum left wheel angular velocity, $\omega_{il,max}(t)$, and the maximum right wheel angular velocity, $\omega_{ir,max}(t)$ are governed by its actuation power constraint [14],

$$\begin{bmatrix} \omega_{il,max}(t) \\ \omega_{ir,max}(t) \end{bmatrix} = \frac{1}{\tau_{roll}} \begin{bmatrix} P_{il,max} \\ P_{ir,max} \end{bmatrix}, \tag{4}$$

where $P_{il,max}$ and $P_{ir,max}$ are maximum power of the robot left wheel and right wheel, τ_{roll} is the wheel rolling resistance from the ground. When the robot traverses, $\omega_{il}(t) \leq \omega_{il,max}(t)$ and $\omega_{ir}(t) \leq \omega_{ir,max}(t)$.

If the left wheel and right wheel power factors $PF_l(t)$ and $PF_r(t)$ are defined as,

$$\begin{bmatrix} PF_l(t) \\ PF_r(t) \end{bmatrix} = \tau_{roll} \begin{bmatrix} \frac{\omega_{il}(t)}{P_{il,max}} \\ \frac{\omega_{ir}(t)}{P_{ir,max}} \end{bmatrix}, \tag{5}$$

the control effort from the proposed controllers in following section should satisfy the power constraints

$$-1 \leq PF_l(t) \leq 1, \quad -1 \leq PF_r(t) \leq 1. \tag{6}$$

IV. Controller Design

In this section, the guidance function based decentralized controller is designed for robots to perform rendezvous while considering network connectivity, collision avoidance, and actuation power constraints.

A. Guidance Function Design

The control strategy is to design a guidance function for a leader robot, which creates a trajectory to the desired destination with the desired configuration. The follower robots aim to achieve consensus with the leader robot and maintain network connectivity by using only local interaction with neighboring robots. Following the work in [15], the guidance function is designed for the leader robot $i \in \mathcal{L}_L$ as $\delta_i^l(t) : \mathcal{S} \to [0, 1)$,

$$\delta_i^l = \frac{1}{\left(1 + H_d \cdot \beta_d / \gamma_d^{\alpha}\right)^{1/\alpha}}, \tag{7}$$

where $\alpha \in \mathbb{R}^+$ is a tuning parameter. H_d is a repulsive potential factor, which is used to achieve the desired final orientation. It is designed as

$$H_d = \varepsilon_{nh} + \left((p_i - p^*)^T \cdot \begin{bmatrix} \cos(\theta^*) \\ \sin(\theta^*) \end{bmatrix} \right)^2, \tag{8}$$

where ε_{nh} is a small positive constant. γ_d is a position factor, which is used to achieve the desired final position. It is designed as

$$\gamma_d = \|p_i(t) - p^*\|^2. \tag{9}$$

Assume each robot i has a collision region defined as a small disk with radius $r_1 < R$, and an escape region defined as the outer ring of the sensing area centered at the node with radius r, $R - r_2 < r < R$, where $r_2 \in \mathbb{R}^+$ is a predetermined buffer distance. To prevent a potential collision between node i and the workspace boundary, the function $\beta_d : \mathbb{R}^2 \to (0, 1)$ in (7) is designed as

$$\beta_d = \frac{1}{1 + e^{-\frac{2}{r_1} \log\left(\frac{1-\epsilon}{\epsilon}\right)\left(d_{i0} - \frac{1}{2} r_1\right)}}, \tag{10}$$

where $0 < \epsilon \ll 1$ is a positive constant, and $d_{i0} \triangleq R_w - \|p_i\| \in \mathbb{R}$ is the relative distance of node i to the workspace boundary.

For the follower robots, in order to achieve the consensus with the leader robot while ensuring network connectivity and collision avoidance, a local interaction rule is designed for each follower robot $i \in \mathcal{L}_F$ as $\delta_i^f(t) : \mathcal{S} \to [0, 1)$,

$$\delta_i^f = \frac{1}{\left(1 + \beta_i / \gamma_i^{\alpha}\right)^{1/\alpha}}, \tag{11}$$

where $\alpha \in \mathbb{R}^+$ is a tuning parameter. The goal function $\gamma_i(t) : \mathbb{R}^2 \to \mathbb{R}^+$ in (11) encodes the control objective of achieving consensus on the position between node i and neighboring nodes $j \in \mathcal{N}_i$, which is designed as

$$\gamma_i = \sum_{j \in \mathcal{N}_i} \|p_i(t) - p_j(t)\|^2. \tag{12}$$

Any node $j \in \mathcal{N}_i$ inside the collision region has the potential to collide with node i, and each edge formed by node i and $j \in \mathcal{N}_i$ in the escape region has the potential to break connectivity. To ensure collision avoidance and network connectivity, the constraint function $\beta_i : \mathbb{R}^{2N} \to (0, 1)$ in (11) is designed as

$$\beta_i = \prod\nolimits_{j \in \mathcal{N}_i} b_{ij} B_{ij}, \tag{13}$$

by only accounting for nodes within its sensing area. Particularly, $b_{ij}(p_i, p_j) : \mathbb{R}^2 \to (0, 1)$ in (13) is a continuously differentiable sigmoid function, designed as

$$b_{ij} = \frac{1}{1 + e^{-\frac{2}{r_2} \log\left(\frac{1-\epsilon}{\epsilon}\right)\left(R - \frac{1}{2} r_2 - d_{ij}\right)}}, \tag{14}$$

where $0 < \epsilon \ll 1$ is a positive constant. The designed b_{ij} ensures connectivity of nodes i and its neighboring nodes $j \in \mathcal{N}_i$ (i.e., nodes $j \in \mathcal{N}_i$ will never leave the sensing and communication zone of node i if node j is initially connected to node i).

Since collision avoidance among robots are only required in S_c, $B_{ij}(p_i,\ p_j) : \mathbb{R}^2 \to (0,1)$ in (13) is designed as

$$B_{ij} = \frac{1}{1 + e^{-\frac{2}{r_1}\log\left(\frac{1-\epsilon}{\epsilon}\right)\left(d_{ij} - \frac{1}{2}r_1\right)}}, \tag{15}$$

which indicates that collision avoidance is activated if the robots are in S_c, i.e., node i is repulsed from other nodes to prevent a collision in S_c. If the robots are in S_r, the collision avoidance is deactivated by removing B_{ij} from β_i in (13).

B. Decentralized Controller Design

In the following controller design process, δ_i has been adopted to represent the guidance function designed for each node i, where particularly $\delta_i = \delta_i^l$ in (7) if $i \in \mathcal{L}_L$, and $\delta_i = \delta_i^f$ in (11) if $i \in \mathcal{L}_F$.

The partial derivative of the guidance function δ_i with respect to p_i is,

$$\nabla_i \delta_i = \left[\begin{array}{cc} \frac{\partial \delta_i}{\partial x_i} & \frac{\partial \delta_i}{\partial y_i} \end{array} \right]^T. \tag{16}$$

The desired orientation for any robot $i \in \mathcal{L}$, denoted by $\theta_{di}(t)$, is defined as a function of the negative gradient of the guidance function δ_i as,

$$\theta_{di} \triangleq \arctan 2\left(-\frac{\partial \delta_i}{\partial y_i},\ \ -\frac{\partial \delta_i}{\partial x_i} \right), \tag{17}$$

where the mapping $\arctan 2(\cdot) : \mathbb{R}^2 \to \mathbb{R}$ denotes the four quadrant inverse tangent function, and $\theta_{di}(t)$ is confined to the region of $(-\pi, \pi]$. By defining $\theta_{di}|_{p^*} = \arctan 2(0,0) = \theta_i|_{p^*}$, θ_{di} remains continuous along any approaching direction to the goal position. Based on the definition of θ_{di} in (17)

$$\nabla_i \delta_i = -\|\nabla_i \delta_i\| \left[\begin{array}{cc} \cos(\theta_{di}) & \sin(\theta_{di}) \end{array} \right]^T, \tag{18}$$

where $\|\nabla_i \delta_i\|$ denotes the Euclidean norm of $\nabla_i \delta_i$. The difference between the current orientation and the desired orientation for robot i at each time instant is defined as

$$\tilde{\theta}_i(t) = \theta_i(t) - \theta_{di}(t). \tag{19}$$

Without considering the wheel power constraint, the control effort for the robot left and right wheel velocities, $\omega_{il}(t)^*$ and $\omega_{ir}(t)^*$, is designed as

$$\omega_{il}(t)^* = \frac{1}{r}(k_{v,i}\|\nabla_i \delta_i\| \cos\tilde{\theta}_i + \frac{k_{w,i}B}{2}\tilde{\theta}_i - \frac{B}{2}\dot{\theta}_{di}), \tag{20}$$

$$\omega_{ir}(t)^* = \frac{1}{r}(k_{v,i}\|\nabla_i \delta_i\| \cos\tilde{\theta}_i - \frac{k_{w,i}B}{2}\tilde{\theta}_i + \frac{B}{2}\dot{\theta}_{di}), \tag{21}$$

where $k_{v,i},\ k_{w,i} \in \mathbb{R}^+$ denote the control gains for robot i. The term $\dot{\theta}_{di}$ in (20) and (21) is determined as

$$\dot{\theta}_{di} = k_{v,i}\cos(\tilde{\theta}_i) \left[\begin{array}{c} \sin(\theta_{di}) \\ -\cos(\theta_{di}) \end{array} \right]^T \nabla_i^2 \delta_i \left[\begin{array}{c} \cos(\theta_i) \\ \sin(\theta_i) \end{array} \right], \tag{22}$$

where $\nabla_i^2 \delta_i$ denotes the Hessian matrix of δ_i with respect to p_i.

The maximum power factor from the left wheel and right wheel is defined as $PF_{max}(t)$. Based on (5), it is described as

$$PF_{max}(t) = \max\{|PF_l(t)^*|, |PF_r(t)^*|\} \tag{23}$$

where,

$$\left[\begin{array}{c} PF_l(t)^* \\ PF_r(t)^* \end{array} \right] = \tau_{roll} \left[\begin{array}{c} \frac{\omega_{il}(t)^*}{P_{il,max}} \\ \frac{\omega_{ir}(t)^*}{P_{ir,max}} \end{array} \right]. \tag{24}$$

Taking into account of the wheel power constraint based on (23), the controller is design as

$$\omega_{il}(t) = \frac{1}{rPS(t)}(k_{v,i}\|\nabla_i \delta_i\| \cos\tilde{\theta}_i + \frac{k_{w,i}B}{2}\tilde{\theta}_i - \frac{B}{2}\dot{\theta}_{di}), \tag{25}$$

$$\omega_{ir}(t) = \frac{1}{rPS(t)}(k_{v,i}\|\nabla_i \delta_i\| \cos\tilde{\theta}_i - \frac{k_{w,i}B}{2}\tilde{\theta}_i + \frac{B}{2}\dot{\theta}_{di}), \tag{26}$$

where $PS(t)$ is the power saturation factor. It is,

$$PS(t) = \begin{cases} 1, & PF_{max}(t) < 1 \\ PF_{max}(t), & PF_{max}(t) \geq 1 \end{cases} \tag{27}$$

V. Connectivity and Convergence Analysis

A. Connectivity Analysis

Theorem 1: *Given an initial graph $\mathcal{G}(0)$ containing a connected spanning tree, the controller in (25) and (26) preserves its connectivity and avoids collision for robots with wheel kinematics model in (3).*

proof: To show every existing edge in the directed spanning tree in $\mathcal{G}(0)$ is preserved, consider a follower $i \in \mathcal{L}_F$ located at a position that causes β_i to approach 0, which will be true when either only one node j is about to disconnect from node i or when multiple nodes are about to disconnect with node i simultaneously. If β_i approaches 0, the guidance function δ_i designed in (11) will achieve its maximum value.

Substituting (25) and (26) into (3) and using the fact that $\left[\begin{array}{cc} \cos\theta_i & \sin\theta_i \end{array} \right] \nabla_i \delta_i = -\|\nabla_i \delta_i\| \cos\tilde{\theta}_i$ from (18), the robot i coordinates in workspace $\mathcal{S}$ can be obtained as

$$\dot{p}_i(t) = -\frac{k_{v,i}}{PS(t)}\nabla_i \delta_i,\ \ i \in \mathcal{L}. \tag{28}$$

Driven by the negative gradient of δ_i in (28), no open set of initial conditions can be attracted to the maxima of the guidance function [4]. Therefore, every edge in $\mathcal{G}$ is maintained and the directed spanning tree structure is preserved for all time.

Similar to the proof of the preservation of each link, if two nodes i and j are about to collide in S_c, that is $B_{ij}(p_i,\ p_j) \to 0$ from (15), then the potential function δ_i in (11) will reach its maximum. Based on the properties of a guidance function driven by (28), the system will not achieve its maximum. Hence, collision among nodes is avoided.

B. Convergence Analysis

Theorem 2: *If the graph $\mathcal{G}$ has a spanning tree with the leader robot as the root and each robot has the wheel kinematic model (3), the controller in (25) and (26) ensures that all robots in S_r converge to a common point with a desired orientation, in the sense that $p_i(t) \to p^*, \tilde{\theta}_i(t) \to 0$ as $t \to \infty\ \forall i \in V$.*

proof: For the leader robot $i \in \mathcal{L}_L$, its position can be described from (28),

$$\dot{p}_i(t) = -\frac{k_{v,i}}{PS(t)}\nabla_i \delta_i, i \in \mathcal{L}_L \tag{29}$$

For the follower robots $i \in \mathcal{L}_F$, starting from (11), $\nabla_i \delta_i$ can be calculated as

$$\nabla_i \delta_i = \frac{\alpha \beta_i \nabla_i \gamma_i - \gamma_i \nabla_i \beta_i}{\alpha (\gamma_i^\alpha + \beta_i)^{\frac{1}{\alpha}+1}}, \tag{30}$$

where $\nabla_i \gamma_i$ and $\nabla_i \beta_i$ can be determined as

$$\nabla_i \gamma_i = 2 \sum\nolimits_{j \in \mathcal{N}_i} (p_i - p_j), \tag{31}$$

and

$$\nabla_i \beta_i = \sum\nolimits_{j \in \mathcal{N}_i} \left(\frac{\partial b_{ij}}{\partial d_{ij}} \right) \frac{\prod_{l \in \mathcal{N}_i, l \neq j} b_{il}}{\|p_i - p_j\|} (p_i - p_j). \tag{32}$$

The term $\frac{\partial b_{ij}}{\partial d_{ij}}$ in (32) is

$$\frac{\partial b_{ij}}{\partial d_{ij}} = -\frac{\frac{2}{r_2} \log\left(\frac{1-\epsilon}{\epsilon}\right) e^{-\frac{2}{r_2} \log\left(\frac{1-\epsilon}{\epsilon}\right)\left(R - \frac{1}{2} r_2 - d_{ij}\right)}}{\left(1 + e^{-\frac{2}{r_2} \log\left(\frac{1-\epsilon}{\epsilon}\right)\left(R - \frac{1}{2} r_2 - d_{ij}\right)}\right)^2}, \tag{33}$$

which is negative. Substituting (31) and (32) into (30), $\nabla_i \delta_i$ is rewritten as

$$\nabla_i \delta_i = \sum\nolimits_{j \in \mathcal{N}_i} m_{ij} (p_i - p_j), \tag{34}$$

where

$$m_{ij} = \frac{2\alpha \beta_i - \left(\frac{\partial b_{ij}}{\partial d_{ij}}\right) \frac{\prod_{l \in \mathcal{N}_i, l \neq j} b_{il}}{\|p_i - p_j\|} \gamma_i}{\alpha (\gamma_i^\alpha + \beta_i)^{\frac{1}{\alpha}+1}} \tag{35}$$

is non-negative, based on the definitions of γ_i, β_i, α, and $\frac{\partial b_{ij}}{\partial d_{ij}}$ in (33). Using (28) and (34), the position of the follower robots can be described as,

$$\dot{p}_i(t) = -\sum_{j \in \mathcal{N}_i} \frac{k_{v,i}}{PS(t)} m_{ij} (p_i - p_j), i \in \mathcal{L}_F. \tag{36}$$

Combining (29) and (36), the position of the entire robot group can be described as

$$\dot{\mathbf{p}}(t) = -(\pi(t) \otimes I_2) \mathbf{p}(t) + \mathbf{F}_d, \tag{37}$$

where $\mathbf{p}(t) = \begin{bmatrix} p_1^T, & \cdots, & p_N^T \end{bmatrix}^T \in \mathbb{R}^{2N}$ denotes the stacked vector of p_i, $\mathbf{F}_d = \begin{bmatrix} -\frac{k_{v,i}}{PS(t)} \nabla_i^T \delta_i^l, & 0, & \cdots, & 0 \end{bmatrix}^T \in \mathbb{R}^{2N}$ for $i = 1$, I_2 is a 2×2 identity matrix, and the elements of $\pi(t) \in \mathbb{R}^{N \times N}$ are defined as

$$\pi_{ik}(t) = \begin{cases} \sum_{j \in \mathcal{N}_i} \frac{k_{v,i}}{PS(t)} m_{ij}, & i = k \\ -\frac{k_{v,i}}{PS(t)} m_{ik}, & k \in \mathcal{N}_i, i \neq k \\ 0, & k \notin \mathcal{N}_i, i \neq k. \end{cases} \tag{38}$$

Since it is known from (35) that m_{ij} is non-negative and $k_{v,i}$ is a positive constant gain, the off-diagonal elements of $\pi(t)$ are negative or zero, and its row sums are zero. Hence, $\pi(t)$ is a Laplacian matrix. Since the leader robot acts as the root in the spanning tree structure in $\mathcal{G}$, the first row of $\pi(t)$ is comprised of all zeros, which indicates that the motion of the leader robot is not dependent upon the motion of the followers. From the properties of the dipolar guidance function in (7), the first term in (37) indicates consensus that $p_1 = \cdots = p_N$, and the second term implies that $p_1 \to p^*$, and hence, $p_i \to p^* \ \forall \ i \in \mathcal{L}$.

It is seen that the properties of the guidance function in (7) ensure that the leader robot achieves the specified destination with the desired orientation. If the leader robot always tracks its desired orientation θ_{di} and all the followers move along with the leader robot, the networked robot system will achieve the destination with desired orientation.

Taking the time derivative of (19), the open-loop orientation tracking error is $\dot{\tilde{\theta}}_i = \omega_i - \dot{\theta}_{di}$. Using (25), (26) and (3), the closed-loop orientation tracking error is

$$\dot{\tilde{\theta}}_i = -\frac{k_{w,i}}{PS(t)} \tilde{\theta}_i, \tag{39}$$

which has a decaying solution, hence $\tilde{\theta}_i \to 0$.

VI. Simulation

Numerical simulation results are provided to demonstrate the performance of the controller developed in (25) and (26) in a scenario in which a networked robot system achieves rendezvous at the common destination $p^* = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ with the desired orientation $\theta^* = 0$. The workspace $\mathcal{S}$ is a disk area centered at the origin with radius $R_w = 50\ m$. The rendezvous region is defined as a disk area centered at the origin with radius $R_r = 1.5\ m$ and the rest of the area in $\mathcal{S}$ is the collision-free region. Each robot is chosen and configured identically. The limited communication and sensing zone for each robot is assumed as $R = 2\ m$ and $r_1 = r_2 = 0.4\ m$. The tuning parameter α in (7) is selected to be $\alpha = 1.2$. The control gains are selected as $k_{w,i} = k_{v,i} = 1.1$ for $\forall i \in \{1, \ldots, 7\}$ and the parameters are set as $\epsilon = 0.01$ and $\varepsilon_{nh} = 0.1$. The robot width is $B = 0.4\ m$. The wheel rolling resistance with ground is $\tau_{roll} = 0.4\ Nm$. The wheel radius is $0.2\ m$. The maximum power of each wheel is $2\ W$.

The networked robot system consists of seven robots. The initial tree formed by the mobile robots is assumed to contain a spanning tree, where the leader robot acts as the root. Table I describes the connectivity of the networked robot system which needs to be preserved during the rendezvous. The top row and left column represents the robot id. In Table I, 1 represents two robots are connected, and 0 represents they are not connected. The connectivity structure of Table I is created by following the rules: (1) if two robots' distance is smaller than R, the two robots' intersection is 1; (2) if two robots' distance is larger than R, the two robots' intersection is 0. If initially every distance between two robots is larger than R, Table I has only 0s. It means that initially no two robots are connected, which is against the assumption in Section II.

TABLE I
NETWORKED ROBOT SYSTEM CONNECTIVITY

robot	#1	#2	#3	#4	#5	#6	#7
#1	0	1	0	0	0	1	0
#2	1	0	1	1	0	0	0
#3	0	1	0	1	0	0	0
#4	0	1	1	0	0	0	0
#5	0	0	0	0	0	1	1
#6	1	0	0	0	1	0	1
#7	0	0	0	0	1	1	0

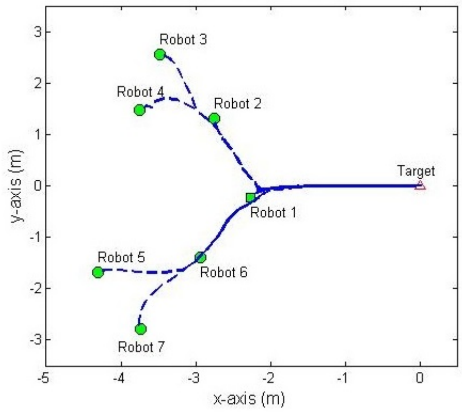

Fig. 2. All robot trajectories to the common destination.

Fig. 2 shows the result of each robot's trajectory to the common destination. The leader robot is represented as square and the follower robots are represented as dots. It clearly shows that the proposed controllers can effectively converge all the robots to the specified destination.

Fig. 3 shows the result of the actual power of the left and right wheel of each robot. Given the power limit of each wheel is 2 W, Fig. 3 shows the control effort is restricted within the actuator capabilities. At $t = 37$ s, small spikes can be observed. It means at this moment the robots transit from the collision region S_c to the rendezvous region S_r and there is no constraint for robot collision avoidance anymore.

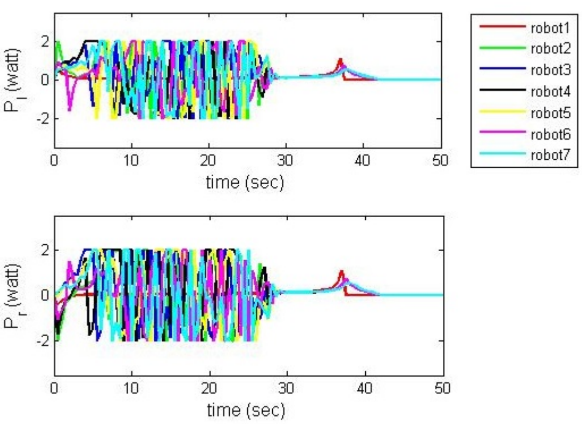

Fig. 3. Power of the left and right wheel of each robot.

The position and orientation errors of each robot are shown in Fig. 4, which indicates that all robots converge to the common destination with desired orientation.

VII. Conclusion

Decentralized controllers for robot wheel velocities are developed to navigate a network of mobile robots to a common destination with a desired orientation while ensuring network connectivity and collision avoidance and also satisfying the robot sensing constraint and robot wheel actuation constraint. One distinguishing feature of the developed approach from the existing research the adoption of the robot wheel level kinematics, which enables control design to take into account the wheel actuation power constraint. The future work will investigate multiple leader robots with their follower robots operating over dynamic network topology.

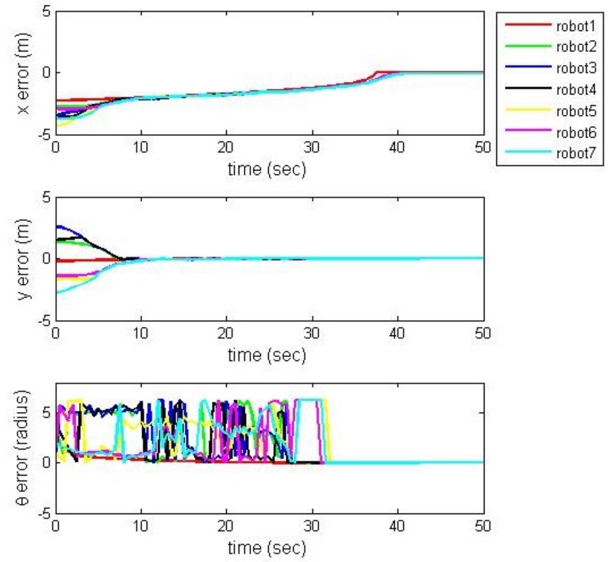

Fig. 4. Plot of position and orientation error for each mobile robot.

References

[1] Lin, J. and Morse, A.S. and Anderson, B.D.O., "The multi-agent rendezvous problem. Part 1: The synchronous case", SIAM J. Control Optim, 2007, pp 2096-2119

[2] Lin, J. and Morse, AS and Anderson, BDO, "The multi-agent rendezvous problem. Part 2: The asynchronous case", SIAM J. Control Optim., 2007, pp 2120-2147

[3] Loizou, S.G. and Kyriakopoulos, K.J., "Navigation of Multiple Kinematically Constrained Robots", IEEE Trans. Robot, 2008, pp 221-231

[4] Rimon, E. and Koditschek, D.E., "Exact robot navigation using artificial potential functions", IEEE Trans. Robot. Autom., 1992, pp 501-518

[5] Roussos, G.P. and Dimarogonas, D.V. and Kyriakopoulos, K.J., "3D navigation and collision avoidance for a non-holonomic vehicle", Proc. Am. Control Conf., 2008, pp 3512-3517

[6] Cort'es, J. and Mart'nez, S. and Bullo, F., "Robust rendezvous for mobile autonomous agents via proximity graphs in arbitrary dimensions", IEEE Trans. Autom. Control, 2006, pp 1289-1298

[7] M. Zavlanos and G. Pappas, "Potential fields for maintaining connectivity of mobile networks", IEEE Trans. Robot., 2007, pp 812?816

[8] Li, Xiangpeng and Sun, Dong and Yang, Jie, "Preserving multirobot connectivity in rendezvous tasks in the presence of obstacles with bounded control input", IEEE Trans. Control Syst. Technol., 2013, pp 2306-2314

[9] Feng Xiao and Long Wang and Tongwen Chen, "Connectivity preservation for multi-agent rendezvous with link failure", Automatica, 2012, pp 25-35

[10] Hui, Q., "Finite-time rendezvous algorithms for mobile autonomous agents", IEEE Trans. Autom. Control, 2011, pp 207-211

[11] Dimarogonas, D. V. and Kyriakopoulos, K. J., "On the Rendezvous Problem for Multiple Nonholonomic Agents", IEEE Trans. Autom. Control, 2007, pp 916-922

[12] Z. Kan and A. Dani and J. Shea and W. E. Dixon, "Ensuring Network Connectivity for Nonholonomic Robots During Rendezvous", Proc. IEEE Conf. Decis. Control, 2011, pp 2369-2374

[13] Z. Kan and J. Klotz and T.-H. Cheng and W. E. Dixon, "Ensuring Network Connectivity for Nonholonomic Robots During Decentralized Rendezvous", Proc. Am. Control Conf., 2012, pp 3718-3723

[14] Wei Yu, Oscar Ylaya Chuy Jr, Emmanuel G Collins Jr, Patrick Hollis, "Analysis and experimental verification for dynamic modeling of a skid-steered wheeled vehicle", IEEE transactions on robotics, 2010, pp 340-353

[15] Tanner, H.G. and Loizou, S.G. and Kyriakopoulos, K.J., "Nonholonomic navigation and control of cooperating mobile manipulators", IEEE Trans. Robot. Autom., 2003, pp 53-64